Example 1

Spring in series : Static and normal modes analysis

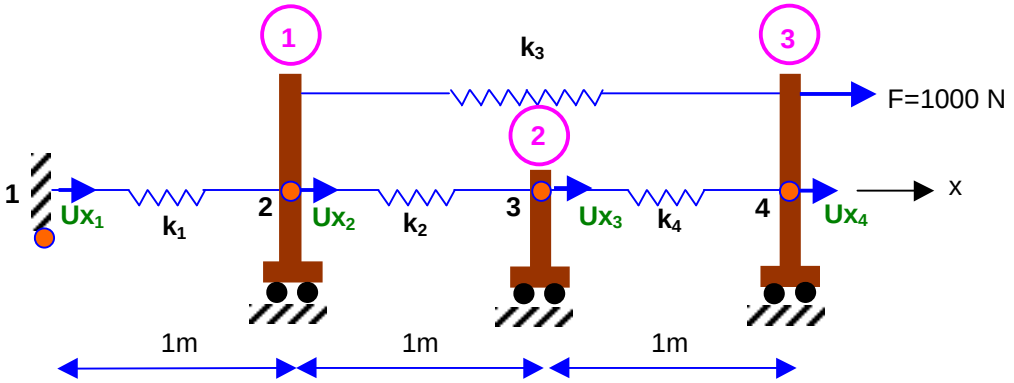

Figure 1 Springs in series

Case 1: The springs in series as shown in Figure 1 are connected to massless rigid bodies which are rolling on a frictionless surface. Assume following spring stiffness : k_1 = 400 N/mm, k_2 = 300 N/mm, k_3 = 500 N/mm and k_4 = 100 N/mm. Force F = 1000 N is applied at grid 4 and displacement at node 3, $\mathbf{Ux_3}$ = - 0.5 mm. Calculate the displacement at Nodes 2 and 4.

Case 2: Assuming mass 1 = 100 kg, mass 2 = 20 kg and mass 3 = 50 kg, find out the natural frequencies and mode shapes for the spring mass system. Assume g = 9.81 m/sec^2.

Example 2: Cantilever beam – Static linear problem

A cantilever beam as shown in Figure 2 is fixed on the left hand end and free at the right. The beam has a symmetric cross section and the load is passing through the shear center. The beam is subject to a uniformly distributed load of 4.29 x 10^3 N/m. The beam has a length = 2.54 m, I_{zz} = I_{yy} = 4.162 x 10^{-3} m^4 , A = 1.2 x 10^{-3} m^2, Young's modulus (E) = 211.36 x 10^9 N/m^2, and Poisson's ratio (ν) = 0.3. Ignore the self weight of the beam. Find the nodal displacements, stresses, shear force and bending moment diagrams.

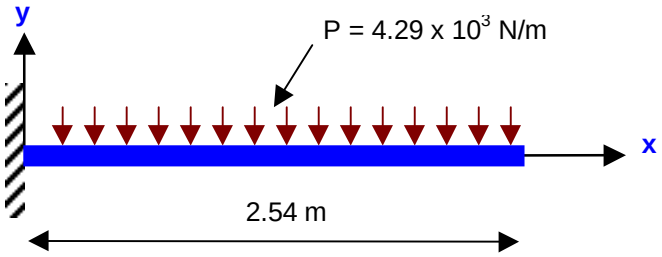

Figure 2 Cantilever beam

Analyze above problem with one-dimensional beam element with two degrees of freedom at each node (translation and rotation) - i). Use one element model and find out displacement & stresses

at both ends of the beam and also at the center of the element. Also find out reaction forces at the fixed end and draw the shear force and bending moment diagram. ii). Use four element model and find out displacement, stresses, and reaction forces. Also draw shear force and bending moment diagrams. and iii). Use ten element model and find out nodal displacement, stresses, and reaction forces. Also draw shear force and bending moment diagrams.

Example 3 – Beam on elastic foundation

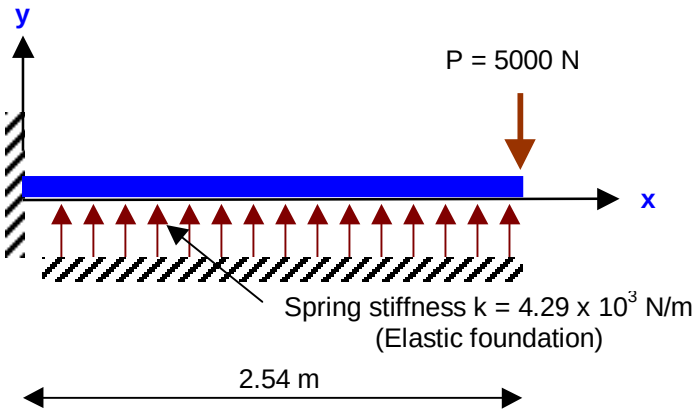

Figure 3 Beam on elastic foundation

A cantilever beam as shown in Figure 3 is resting on the elastic foundation with the stiffness k = 4.29×10^3 N/m. The beam is fixed on the left hand end and is subject to a load of 5000 N at the free end and is supported with the spring stiffness (k), simulating elastic foundation. Calculate the deflection of the beam with length = 2.54 m, $I_{zz} = I_{yy} = 4.162 \times 10^{-3}$ m^4 , A = 1.2×10^{-3} m^2, Young's modulus, E = 211.36×10^9 N/m^2 and Poisson's ratio n = 0.30. Ignore the self weight of the beam.

Solve the problem with one element, two elements, and 10 elements models. Find out nodal displacements, stresses and deflected shape for the beam along with the shear force diagram, the bending moment diagram diagrams, and reaction forces.

Example 4

Cantilever rigid beam with multi-point constraint

A cantilever rigid beam as shown in Figure 4 has a hinge support at the left and supported at nodes 1 and 2 with a steel and aluminum wires. A steel wire is attached between nodes 1 and 3 and has a length = 4.5m, area of 1200 mm^2, and Young's modulus E = 200×10^9 N/mm^2 and Poisson's ratio ν = 0.3. An Aluminum wire is attached between modes 2 and 4 has length = 3m, area of 900 mm^2, and Young's modulus E = 70×10^7 N/mm^2. Multipoint constraints for the nodal

displacements on the rigid beam are defined as : $Uy_1 - 0.33\ Uy_5 = 0$ and $Uy_2 - 0.833\ Uy_5 = 0$, where Uy_1 represents displacement at node 1 in the y-direction, Uy_5 represents y-displacement at node 5, and Uy_2 represents displacement at node 2 in the y-direction. Find out the displacements at nodes 1, 2, and 5 and strain/stresses & forces in element 1 (connecting nodes 1 and 3) and element 2 (connecting nodes 2 and 4). Find out reaction forces at all support points.

Case 1: Model the problem with 2 elements representing the wires and Multi Point Constraints (MPCs) ;

Case 2: Model the problem with a rigid bar between nodes 6, 1, 2, & 5, and 2 wires.

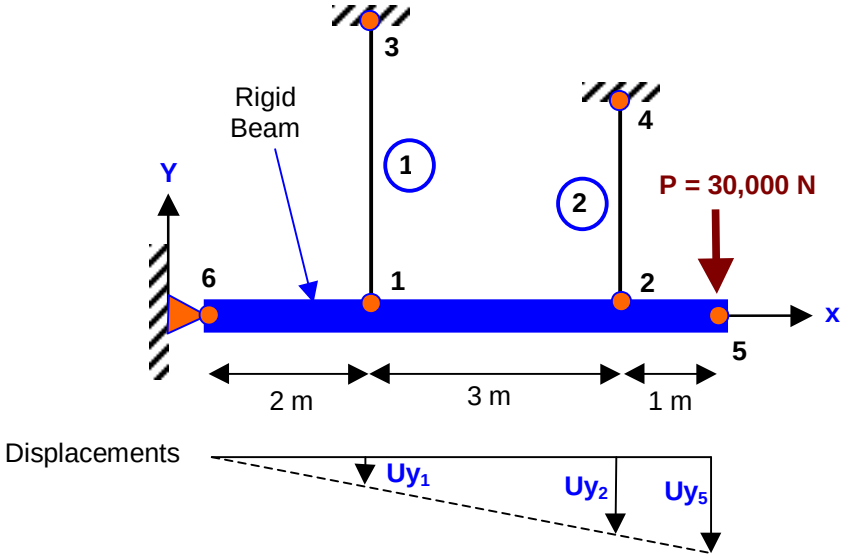

Figure 4 Rigid cantilever beam with multipoint constraint

Example 5

Axial buckling load for a column

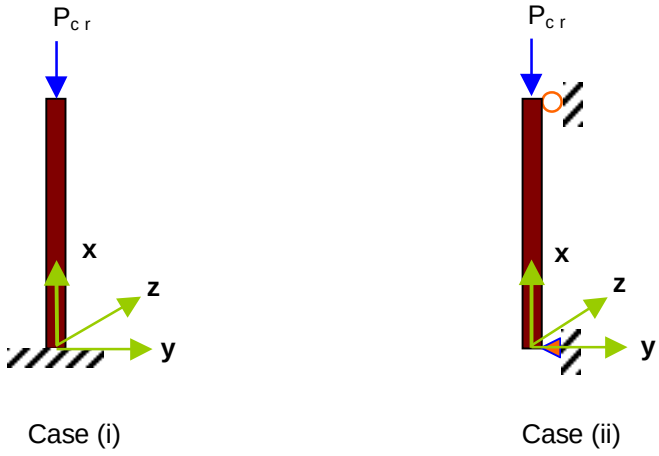

Figure 5 Axial buckling load for a column

For the column shown in Figure 5 calculate the buckling load (i) with the fixed and free boundary condition and (ii) with hinge and roller boundary condition find out the axial buckling load for the column. Assume, length = 2.54 m, $I_{yy} = I_{zz} = 4.162 \times 10^{-3}$ m^4 , A = 1.2×10^{-3} m^2, Young's modulus (E) = 211.36×10^9 N/m^2 and Poisson's ratio (ν) = 0.3 . Ignore the self weight of the column.

Example 6

Heat conduction and convection in a thin fin

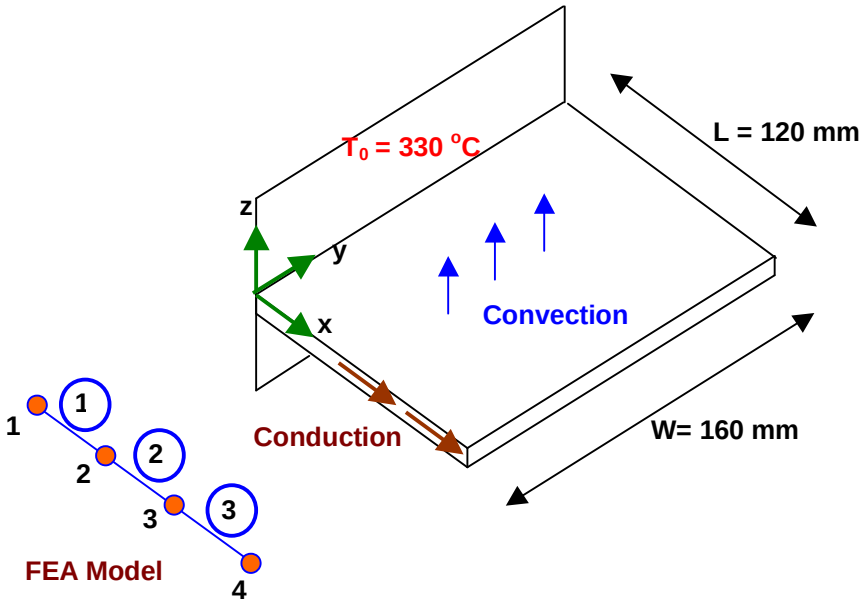

Figure 6 heat conduction and convection in a thin fin

Following parameters are to be used for simulations-

1. Boundary conditions : $T_o = 330^o$ C at x =0 mm, $T_L = 30^o$ C at x = 120 mm
2. $T_{infinity}$ = temperature of fluid surrounding the find, which is lower than the temperature T of the plate = 30^o C (hence the heat convects out of the fin into the fluid);
3. Thickness of the plate = t = 1.25 mm
4. k = Conduction coefficient = 0.2 W/mm^2 ^{o}C; h = convection coefficient = 2×10^{-4} W/mm^2 ^{o}C
4. Cross section area A_x = 160 x 1.25 = 200 mm^2 = W x t

Find out the temperature at node 2 (x = 40 mm) and node 3 (x = 80 mm) due to heat conduction and convection for the following cases assuming boundary conditions same as defined in point number 1 –

1. Conduction problem : Assume convection coefficient h = 0 W/mm^2 ^{o}C i.e. assume that the plate is fully insulated from the top and bottom. Assume conduction coefficient (k) = 0.2 W/mm^2 ^{o}C.

2. Conduction and convection problem : Conduction coefficient (k) = 0.2 W/mm^2 ^{o}C and convection coefficient h = 2 x 10^{-4} W/mm^2 ^{o}C and $T_{infinity}$ = 30^o C.

Example 7

Truss problem

A three member truss as shown in Figure 7 is having hinge support at node 1 and roller support at node 2. The truss members have cross sectional area of 200 mm^2 for elements 1 & 2, and 100 mm^2 for element 3. The truss is subject to a load of P= 400 N at the free end. Young's modulus (E) = 211.36 x 10^9 N/m^2 and Poisson's ratio (ν) = 0.3 . Ignore the self weight of the truss. Find the forces within the elements & reaction forces at the support, and the nodal displacements.

What will be the displacements and reaction forces if the roller support is replaced with the hinge support ?

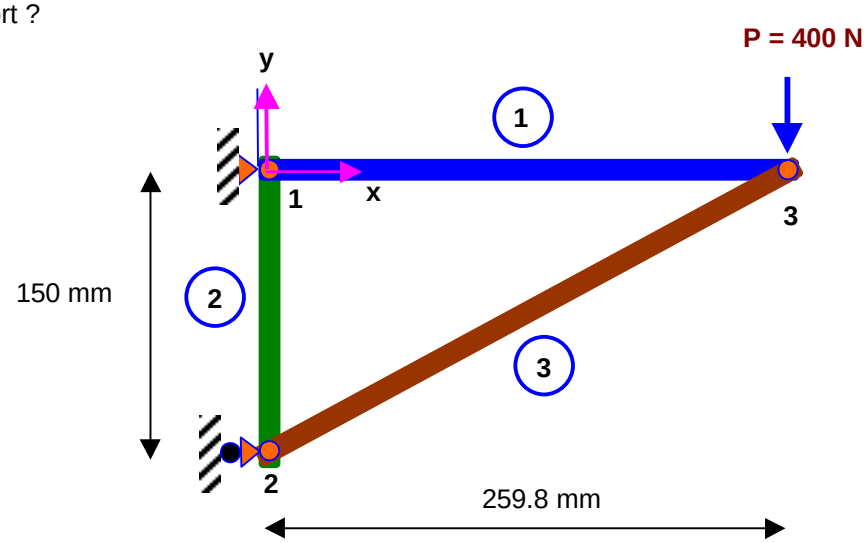

Figure 7 Three member truss

Example 8

Tapered bar with axial displacement

A tapered bar shown in Figure 8 has a diameter of 0.0254 m at the fixed end and has a diameter of 0.0127m at the free end. Assuming initial displacement at node 4 i.e. Ux_4 = 0.038 x 10^{-4} m, Young's modulus (E) for steel = 200 x 10^9 N/mm^2, shear modulus (G) for steel = 70 x 10^3

N/mm^2 and mass density (ρ) (steel) = 7.81 x 10^{-6} kg/mm^3, and Poisson's ratio ν = 0.3. Calculate the displacements at nodes 2 and 3 and stresses in the elements 1, 2, and 3.

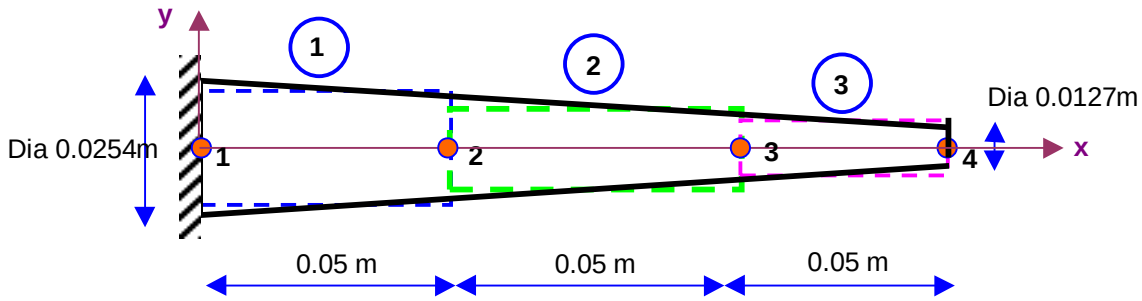

Figure 8 Tapered bar subject to axial load

Example 9

Beam cab model

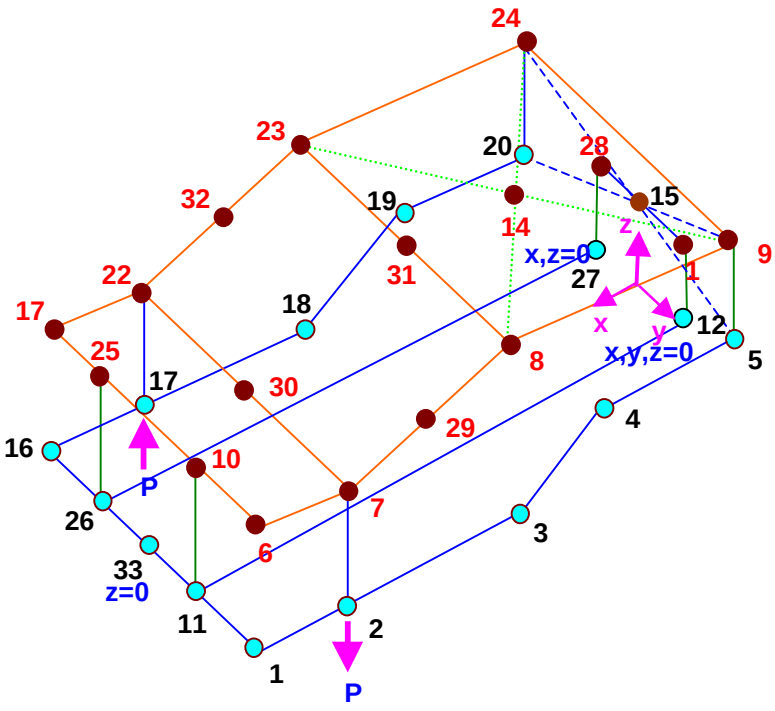

Figure 9 Beam model for a truck cab simulating static torsion

Analyze the truck cab as shown in Figure 9 using beam elements subject to the torsional load of P= 3000 N applied at Nodes 2 and 17. The truck cab structure is constrained at Nodes 33 with displacement U_z = 0; at Node 12 displacements U_x, U_y, U_z = 0, and at Node 27 with displacements U_x, U_z = 0. Find out the defected shape and maximum stresses in the beam elements. Find the nodal displacements, stresses for all grid points and reaction forces at Nodes 33, 12, and 27. Assume following beam properties: A= 237 x 10^{-6} m^2 with uniform section; I_{zz} =

323 x 10^{-9} m^4; and I_{yy} = 111 x 10^{-9} m^4. Assume no offset for the shear center from the node point location. E = 211.36 x 109 N/m^2 and Poisson's ratio n = 0.3.

Nodal locations-

Node	X (mm)	Y (mm)	Z (mm)
1	1473	965	0
2	1219	965	0
3	787	787	0
4	431	965	558
5	0	965	609
6	0	965	1066
7	1473	965	1066
8	914	965	1778
9	0	965	1905
10	1473	431	1066
11	1473	431	0
12	0	431	0
13	0	431	609
14	457	0	1828
15	0	0	952
16	1473	-965	0
17	1219	-965	0
18	787	-787	0
19	431	-965	558
20	0	-965	609
21	1473	-965	1066
22	1219	-965	1066
23	914	-965	1778
24	0	-965	1905
25	1473	-431	1066
26	1473	-431	0
27	0	-431	0
28	0	-431	609
29	1066	965	1422
30	1219	0	1066
31	1219	0	1066
32	1066	-965	1422
33	1473	0	0

Element connectivity

Element Num	Node number	Node number
1	1	2
2	2	3
3	3	4
4	4	5
5	5	9
6	8	9

7	8	29
8	7	6
9	6	10
10	1	11
11	10	11
12	11	12
13	12	13
14	13	28
15	10	25
16	11	33
17	33	26
18	8	31
19	9	24
20	5	15
21	9	14
22	8	14
23	26	27
24	27	28
25	2	7
26	17	22
27	16	17
28	17	18
29	18	19
30	19	20
31	20	24
32	24	23
33	22	32
34	22	21
35	21	25
36	14	23
37	14	24
38	15	24
39	9	15
40	15	20

Example 10

Fluid structure interaction problem

Problem definition:

As shown in Figure 10, a box model with the coupled structure and acoustic cavity is subject to seismic mass input between 0 to 80 hz. The box model is supported by the springs in x, y, and z-directions at the corner. Assume following design parameters for the box model-

i. **Spring rates:** at nodes 1, 2, 3, 4: X-direction = 300 N/mm, Y-direction = 250 N/mm, Z-direction = 500 N/mm

ii. **Load :** Seismic mass = 5,000 kg, g = 9.81 m/sec^2

iii. **Dimensions :** Overall length = 5.0 m, overall width = 2.0 m, overall height 2.5 m, front end height = 1.5m, front end overhang = 2m (please see Figure 10).

iv. **Material properties** : The front windshield is made with the glass with Young's modulus (E) = 2.0 x 10^4 N/mm^2 ; Shear Modulus (G) = 1.0 x 10^4 N/mm^2; density (ρ) = 1.74 x 10^{-6} kg/mm^3 ; Poisson's ratio (ν) = 0.20 . Define joining between the windshield and the body panel with springs in x, y, and z-directions. Model connection between the windshield and the body with the springs - perpendicular to glass plane with 6500 N/mm (z-dir) and in shear with 650 N/mm (x & y-dir). Assume all panels made up of steel with (E) = 211.36 x 10^3 N/mm^2 ; shear modulus (G) = 70 x 10^3 N/mm^2 and density (ρ) = 7.81 x 10^{-6} kg/mm^3, Poisson's ratio (ν) = 0.30.

v. **Panel thicknesses:** As shown in Figure 11, panel thickness are : front panel = 0.5mm, front floor = 0.60 mm (y = 0 to y =2m), mid floor = 0.60 mm (between y=2.0 to y=3.5), rear floor = 0.60 mm (y=3.5m to y=5m), rear panel 0.60 mm, roof = 0.60 mm, front windshield (glass) = 6 mm. The lower limit for all panel thicknesses are 0.5 mm and the upper limit is 0.75 mm for optimization. The lower limit for the glass is 5 mm and upper limit is 7 mm.

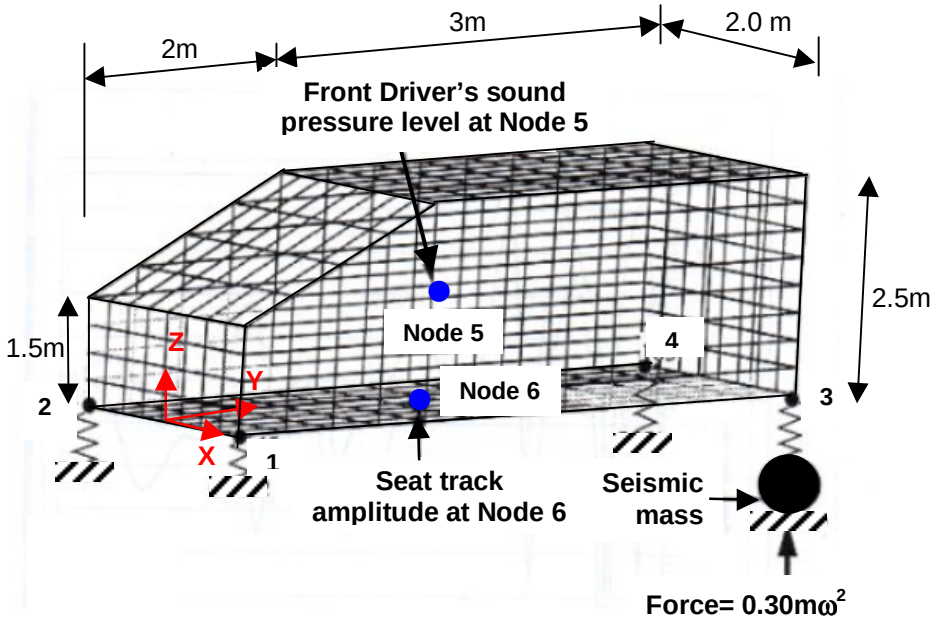

Figure 10 Box model analysis with fluid-structure interaction with frequency response

vi. **Responses :** i. Seat track amplitude (mm) at Node 5 at location x = -0.5 m, y = 2.0 m, z = 0.0 m and ii. Sound pressure (N/mm^2) at Node 6 at location x= - 0.5 m, y = 2.0 m, z = 2.0 m.

vii. **Acoustic model –** Air inside the box to be modelled with acoustic elements having interface with the structure

viii. **Objective :** Reduce the sound pressure level for the highest peak pressure for 2 frequencies separated by 1 Hz adjacent to the peak.

Perform following analysis -

i. Normal mode analysis for the body and find out first 160 frequencies and mode shapes. Identify first bending & torsional mode for the body.

ii. Normal mode analysis for the acoustic cavity and find out first 20 frequencies & mode shapes.

iii. Coupled fluid-structure frequency response analysis with input frequency ω between 0 to 80 Hz with the seismic load as shown in Figure 10 and predict the amplitude of vibration at the seat track i.e. at Node 5 between 0 to 80 Hz and sound pressure level at the Driver's ear level i.e. at Node 6 between 0 to 80 Hz. Consider frequencies upto 160 Hz for performing the frequency response analysis upto 80 Hz.

iv. Design sensitivity analysis for the first bending and torsion mode with respect to all panel thicknesses

v. Design sensitivity analysis for the sound pressure level at Node 5 with respect to panel thicknesses as design variables and vibration amplitude at the seat track at Node 6 for the highest peak within 0 to 80 Hz with respect to panel thicknesses as design variables.

vi. Minimize the peak sound pressure level subject to reducing the mass by 20%.

Example 11

Static torsion simulation for the box model

Analyze the box model for the static torsion load as shown in Figure 11 for the following -

i. **Loads:** Static torsion load applied for 3000 N

ii. **Constraints :** constrain the point at x=0,y=0,z=0 in the z-direction i.e. U_z=0; left rear end of the box model is constraint in x, y, and z-directions U_x=U_y=U_z=0; right rear end constraint in x and z-direction i.e. U_x=U_z=0. Increase the stiffness by 20% compared to the baseline level.

iii. **Analysis :** Static torsion analysis

iv. **Design variables :** 1. Front panel , 2. Front floor, 3. mid floor, 4. Rear floor, 5. Rear panel, 6. Roof, 7. Front windshield. Lower and upper limits for the panel design variables

can be 0.5 mm and 0.75 mm. For the windshield, upper and lower limits are 5 mm and 7 mm.

v. **Response :** Static torsional stiffness of the body measured in N/radian

vi. **Sensitivity analysis :** Find the sensitivity of static torsional stiffness with respect to panel thickneesses for the baseline design

vii. **Optimization** - Optimize the weight subject to the static torsion constraint. Increase the stiffness by 20%

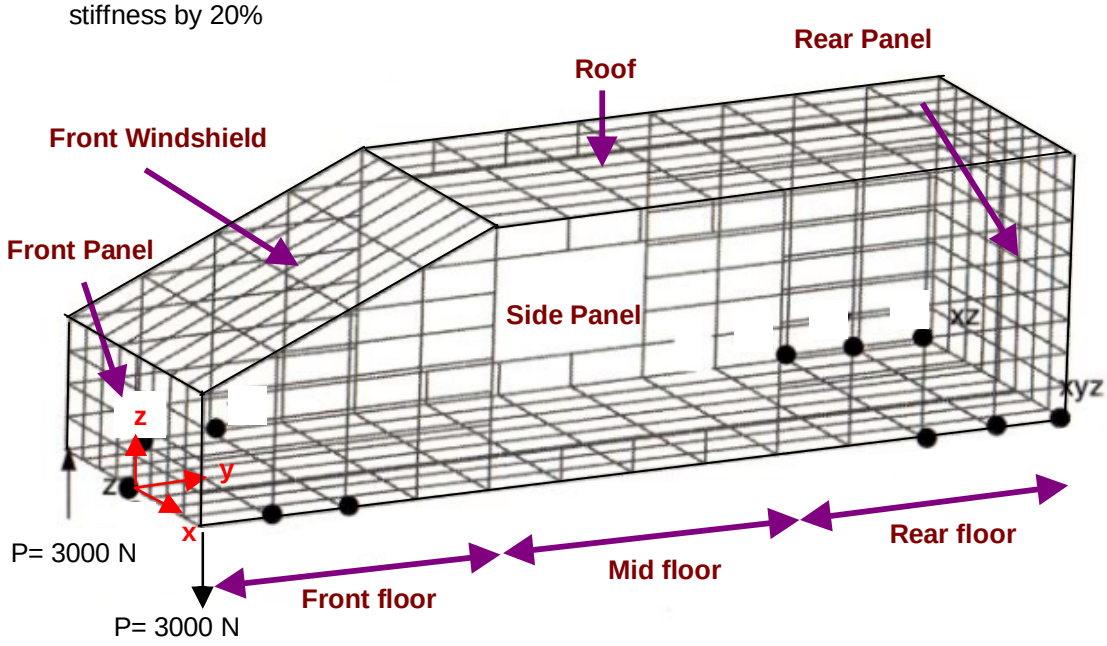

Figure 11 Box model for static torsion analysis

Example 12

Topology Optimization for the box model for torsional stiffness and normal modes:

Case 1 : For the static torsion load case shown in Figure 11, minimize the mass subject to reducing the displacement at the point of application of the load by 20% from the baseline level. Consider following panels as a design space - the front floor, mid floor, rear floor, side panels, front panel, rear panel and roof.

Case 2 : For problem shown in Figure 11, minimize the mass subject to increasing the first torsional frequency by 20%. For topology optimization, consider front floor, mid floor, rear floor, side panels, front panel, rear panel and roof as space.

Example 6

Topology optimization for a symmetric beam subject to load at the center :

Using topology optimization for a 3D symmetric beam shown in Figure 10, minimize the mass and increase the stiffness and first bending frequency.

Material : Steel with Young's modulus E (steel) = 211.36 x 10^3 N/mm^2 ; shear modulus G (steel) = 70 x 10^3 N/mm^2 and density Rho (steel) = 7.81 x 10^{-6} kg/mm^3, Poisson's ratio for steel = 0.30. Initial thickness of the plate is 10 mm.

Objective : Minimize mass

Constraint : Case 1 : Minimize the body weight subject to reducing the displacement at the free end by 25%. Assume entire structure as a design space for topology optimization.

Case 2 : Minimize the body weight subject to increasing the first torsional frequency by 25%. Assume entire structure as a design space for topology.

Design space : Entire plate

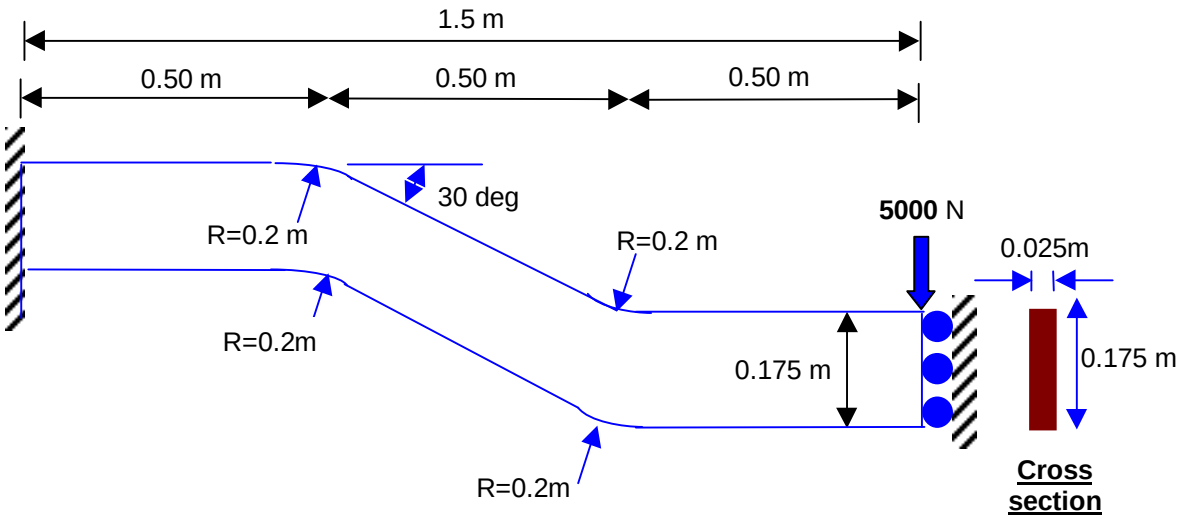

Figure 12 Topology optimization of a beam